Ptolemy's World Map and Eratosthenes's Circumference of the Earth

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Abstract. The interlink between the determination of the circumference of the Earth and the geographical mapping performed by Ptolemy in his *Geography* is discussed. It is shown that a simple transformation of the Ptolemaic coordinates to the (bigger) circumference of the Earth measured by Eratosthenes drastically improves the positions of the locations given in Ptolemy´s catalogue. As a consequence, by linking the recalculated positions of the identified localities with their actual positions, the very high precision of Eratosthenes' result for the circumference of the Earth is confirmed.

Keywords: Ancient Geography, Ptolemy, Eratosthenes

1. Introduction

One of the most surprising features of Ptolemy's world map (ca. 150 AD) is its excessive distortion along the east-west direction. The whole oikoumene from the Fortunate Islands (the Canaries) in the West till the metropolis of the Sines, Kattigara or Sera Metropolis (Xi'an in China?) in the Far East is equivalent to 180 degrees, too large by more than one third. A convincing explanation is still missing. Neither the assertion that Ptolemy subscribed to an aprioristic view that the oikoumene measures exactly half the circumference of the Earth nor the fact that ancient eclipses were found with an interval of 12 hours between both extremities (Ptol. synt. 2, 1, p. 88 Heiberg) of the oikoumene (see, e.g. Stückelberger/Mittenhuber 2009: 262) could have induced Ptolemy to draw such a distorted world map: we hear nothing about such an aprioristic belief in ancient sources (in fact, ancient geographers before Ptolemy have made vastly different calculations and guesses) and Ptolemy himself made clear in his introduction (Geogr.

1,4,1) that he had only a few, if any, reliable observations of eclipses at his disposal and needed to make use of terrestrial measurements recorded in travelogues and itineraries. In fact, the source data Ptolemy had at his disposal was not a table of angles but rather of measured or inferred distances expressed in stades, miles, dayruns and other customary units which he had to convert into arc measures to fit the world map under construction. In such recalculations, the adopted size of the Earth is of primary importance and the question whether Ptolemy used the same definition of a stade as Eratosthenes has fascinated the scholars ever since the rediscovery of Ptolemy's *Geography*. Without standardisation of the metrical units in antiquity, no reliable answer can be found and the question of the resulting circumference of the Earth by Eratosthenes (252,000 stades) or by Ptolemy (180,000 stades) was essentially educated guesswork.[1] Since all the confusing data produced very different (and far from agreed upon) results, we decided to tackle the problem in the following way. Instead of speculating about the modern metrical value of a stadium (or stade) used by ancient scholars, we recalculated the geographical positions given by Ptolemy in his *Geography* assuming that his definition of the stade coincides with the definition of the stade used by Eratosthenes in his estimation of the circumference of the Earth. The comparison of the recalculated Ptolemaic coordinates for a "bigger" Earth with the modern values confirmed our assumption about the equality of the Ptolemy's and Eratosthenes's stade.

2. Recalculation of Ptolemaic coordinates

2.1. Mathematical approach

To treat the problem with the methods of spherical trigonometry, we should assume that sufficiently long historical routes were mostly aligned with the great circles of the Earth's surface. The deviations from these energetically preferable routes due to landscape features or orientation problems can statistically balance each other out at sufficiently large distances. The validity of this presupposition can be checked at the final stage of our analysis in comparison with the real and recalculated positions of localities.

The coordinates of every location are improved relative to a chosen reference point through recalculation of a spherical triangle with the vertices at the localities themselves and the North pole from a sphere with the circumference of 180,000 stades to a sphere with the circumference of 252,000 stades.

[1] On Ptolemy's own measurement see Geus & Tupikova (2013).

The first step of recalculation of the original positions is to restore Ptolemy's raw data, that is, the distances between different localities which he had at his disposal and - in some cases - the directions of the routes connecting these localities. Let us emphasize that without any information about the Earth's size, the exact geographical location is not unambiguously determined by the set of latitudes and pairwise distances. It can be shown that, depending on the Earth's size, different locations at different longitudes can be reached by routes of the same length.

The first procedure can be applied to the places with correctly determined (for example, by astronomical observations) geographical latitudes. For these localities only the longitudes should be corrected. Simple mathematical considerations show that it is due to this case that Ptolemy's world map seems to be elongated along the east-west axis. Among the few reliable data which were available to Ptolemy at his time, the rare latitudinal values of some prominent locations laid the groundwork for Ptolemy´s mapping, the staging of the whole construction. Due to the erroneously adopted size of the Earth, Ptolemy should have consequently obtained a bigger longitudinal difference for every pair of locations with known latitudes and known distance between them (see *Figure 1*, points B and b, respectively).

The second procedure can be applied when recalculating the coordinates of the place lying at a latitude which was unknown to Ptolemy. The geographical position of such a locality must have been calculated by Ptolemy only on the basis of the length of the route and the estimated angle of the route connecting this locality with a starting point with the known latitude. In this case, the geographical latitude as well as the difference in longitudes between two localities was corrected (points C and c in *Figure 1*).

In a special case of localities lying on the same meridian, the positions on the “small” Earth slide in the coordinate grid only along the north-south direction (points D and d in *Figure 1*). The lists of such cities, called *antikeimenoi poleis*, circulated in antiquity since the time of the pre-geographical mapping as a means of a rough orientation between major cardinal points like important cities, ports or landmarks. Although only rarely, the cases of unexpected latitudinal displacement can be observed on Ptolemy's world map. One of these instances is the notorious displacement of Carthage, ca. 4 degrees off in latitude.

The case of a locality lying on the meridian of the reference point with the known latitude **and** with the known distance was much more challenging for Ptolemy. With the angular measure of this distance on the “small” Earth he was not able to reach the known latitude of such a locality. In this case, as a professional astronomer who puts more trust in the astronomical observations, he just would have preferred to transmit the known latitude of a

locality on his map (*Figure 1*, points E and e, respectively) and dismiss the less reliable distance measure. Such cases can be easily detected on Ptolemy's map; from a mathematical point of view, it is more complex to recalculate the position of localities in vicinity of such points with “transmitted” latitudes. Although the positioning of such localities matches very well their actual position, they are strictly speaking “not from this map” and the coordinates of the localities which were adjusted in a local map around such “alien” locations will be generally distorted. A characteristic example is the shape of Sicily distorted due to the transmission of the latitude of Syracuse followed by adjustment of the coast-line points at the distances recalculated by Ptolemy in the angular measure according to his erroneous size of the Earth.

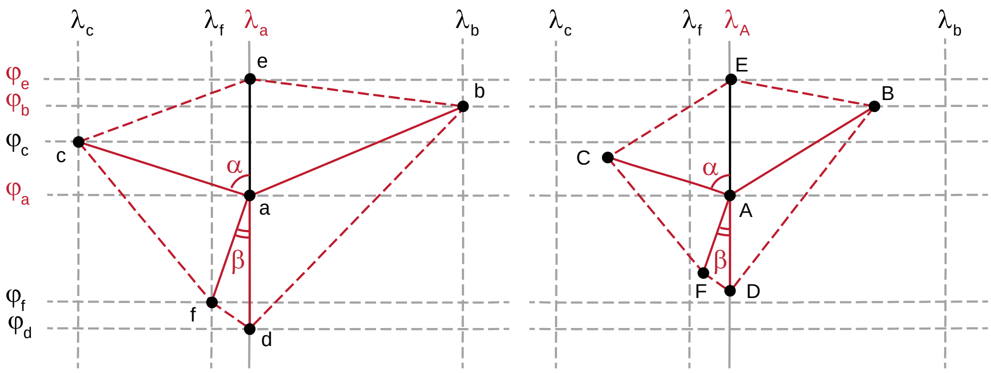

Figure 1. Schematical illustration of the possible cases of distortion in Ptolemy's world map. Left: “small” Earth. Right: “big” Earth. The point a = A is the starting point of the mapping. The known quantities are marked with red color.

2.2. The Problem of the Prime Meridian

Having corrected the positions in relation to any chosen reference point, we face now the problem of comparing the recalculated positions of the localities with their actual geographical location.The positions in Ptolemy's *Geography* are established practically in the modern coordinate system. In fact, it was his treatise where for the first time the spherical equatorial coordinate system was subsequently applied for geographical mapping. Whereas the Ptolemaic latitudes can be considered as being equivalent to the modern values, his longitudinal values should be corrected for the posi-

tion of his Prime Meridian, *Insulae Fortunatae* (*Geogr*. 1,11,1) in relation to the modern Greenwich meridian. The position of Alexandria relative to the *Insulae Fortunatae* is given as 60° 30' (*Geogr*. 4,5,9); the modern longitude of Alexandria is about 29° N 55' E. Subtracting this value from Alexandria's longitude given by Ptolemy one can obtain the longitude of Greenwich relative to Ptolemy' prime meridian as 30° 35'.

It one tries to recalculate the position of Ptolemy's prime meridian through the coordinates of Rome (Ptolemaic position 36° 40', modern position 12° 29'), one obtains 24° 11'. Whatever identified location will be chosen, the position of the Greenwich meridian relative to the *Insulae Fortunatae* will always be different. The problem does not lie in the poor determination of the positions in Ptolemy's time: it lies in Ptolemy's attempt to map the available distances onto a sphere of a wrong size. As a result, all the local regions of Ptolemy's maps are distorted relative to *every* starting point of his distances data. The maps are stretched along the east-west direction for the localities with known latitudes and along all the other possible directions in other cases.

This is why the identification of the position of the Greenwich meridian through the modern coordinates of identified localities is always delicate – it slides along the modern coordinate system. As a consequence, it can not be linked with the Ptolemaic coordinates globally. In our view, it makes no sense to speak of the position of the Greenwich meridian relative to Ptolemy's zero meridian without mentioning the chosen reference point.

In our method, the recalculated coordinates of every point are linked with the position of the Greenwich meridian found through the reference point used in the procedure.

2.3. Recalculated Maps of Mediterranean

In order to make our results "more visible", only some important cities which played a major role as ancient ports and hubs are displayed on our recalculated map of the Mediterranean. In the following illustrations, the black circles with black numbers indicate the modern positions of locations, the red circles with red numbers the positions as given by Ptolemy in his *Geography* referred to the Greenwich longitude determined relative to a selected reference point and the positions recalculated for the "big" Earth's size are marked with yellow circles. We have started first with the recalculation of Ptolemy's coordinates for the eastern part of Mediterranean.[2] It is to

[2] We have taken the coordinates of the Ptolemaic localities from the new edition by Stückelberger & Graßhoff (2006).

be expected that, at this scale, the error in the determination of the Earth's size does not manifest itself so drastically as at the extremities of the *oikoumene*; on the other hand, the positions of the locations were better known and the results can be easily verified.

In order to eliminate the expected longitudinal error of Rome relative to Alexandria, we have chosen for our map Rome's position (strictly speaking, the coordinates of the famous *Milliarium Aureum* at the Forum Romanum) as the reference point and recalculated the Ptolemaic coordinates first for the “Greenwich reduction’’ relative to Rome, 24° 11'. Then, the coordinates were recalculated for the “big’’ Earth. The results given in *Figure 2* show a striking improvement of the Italic positions recalculated for the circumference of the Earth equal to 252,000 stades.

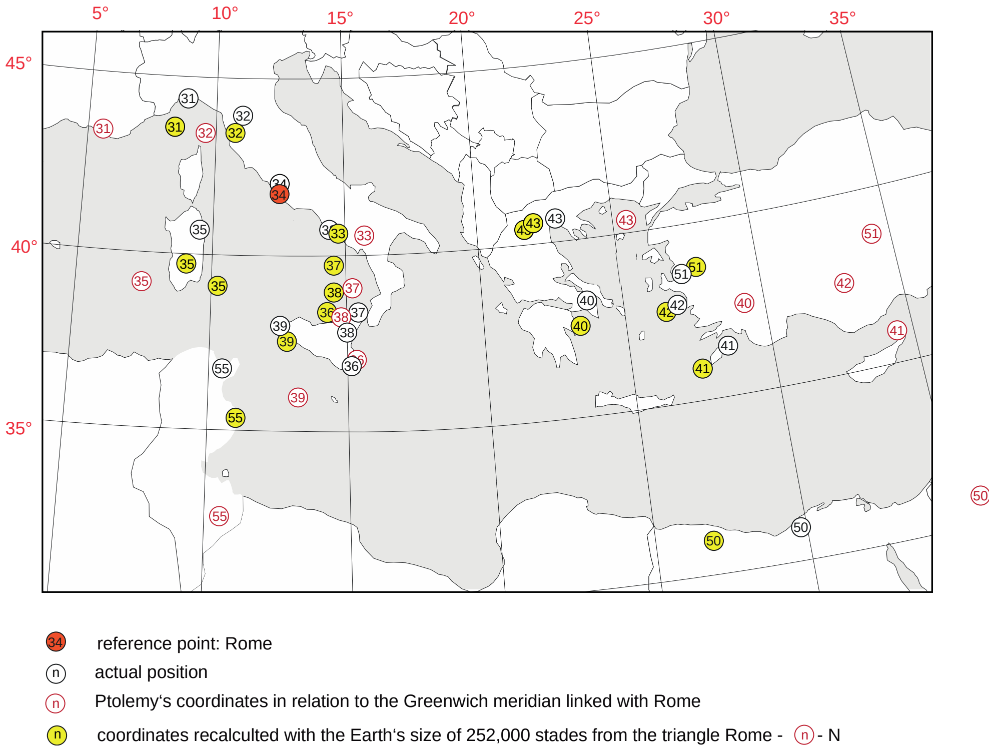

Figure 2. Recalculation of some prominent locations in the eastern Mediterranean for the circumference of the Earth equal to 252,000 stades. The reference point for this recalculation is Rome.

The recalculated Ptolemaic coordinates of Rhodes, Samos and Smyrna can also very well been corrected for their longitudinal displacement. This fact, in our opinion, means that originally their coordinates came from Italian sources and the position of Alexandria was inevitably linked with these localities with well known latitudes and mutual positions. Again, the problem of the erroneous size of the Earth pops up here. If Ptolemy had chosen to preserve the accurate distances to these islands, he should have placed Alexandria far more in the eastern direction as he did in the end. As a result, this part of Mediterranean is not coherent with the rest of the map. Alexandria as well as the localities linked with it, all have wrong longitudinal alignments and should therefore be considered as an own case.

According to the data base for the ancient distances collected by K. Geus,[3] the most cited reference points are in the this order: Rome, Carthage, Mons Calpe, Alexandria and Babylon. Because of that, we have also chosen Rome as a reference point to recalculate the Ptolemaic positions for Spain and Gaul. The preliminary results are shown in *Figure 3*.

Because at such huge distances relative to the reference point the erroneously adopted size of the Earth manifests itself at a bigger scale, the results of the correction for the Earth's size are also much more visible. The distribution of recalculated coordinates shows different patterns in Gaul and Italy. The recalculated locations of Tolosa (Toulouse), Lugdunum Metropolis (Lyon), Burdigala (Bordeaux) and Massilia (Marseille) are of a remarkable precision - it is obvious that their positions were in fact defined in relation to Rome. The coordinates in Spain are also drastically improved but show another displacement's pattern: all the recalculated coordinates lie in the west of the actual positions. Because a small displacement in the same direction shows also the coordinates of Marseille, one can suggest that its location served as a starting point for Ptolemy's mapping of Spain. This aspect merits further investigation.

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[3] This data base on ancient measurements is a project carried out in the excellence cluster TOPOI (Berlin).

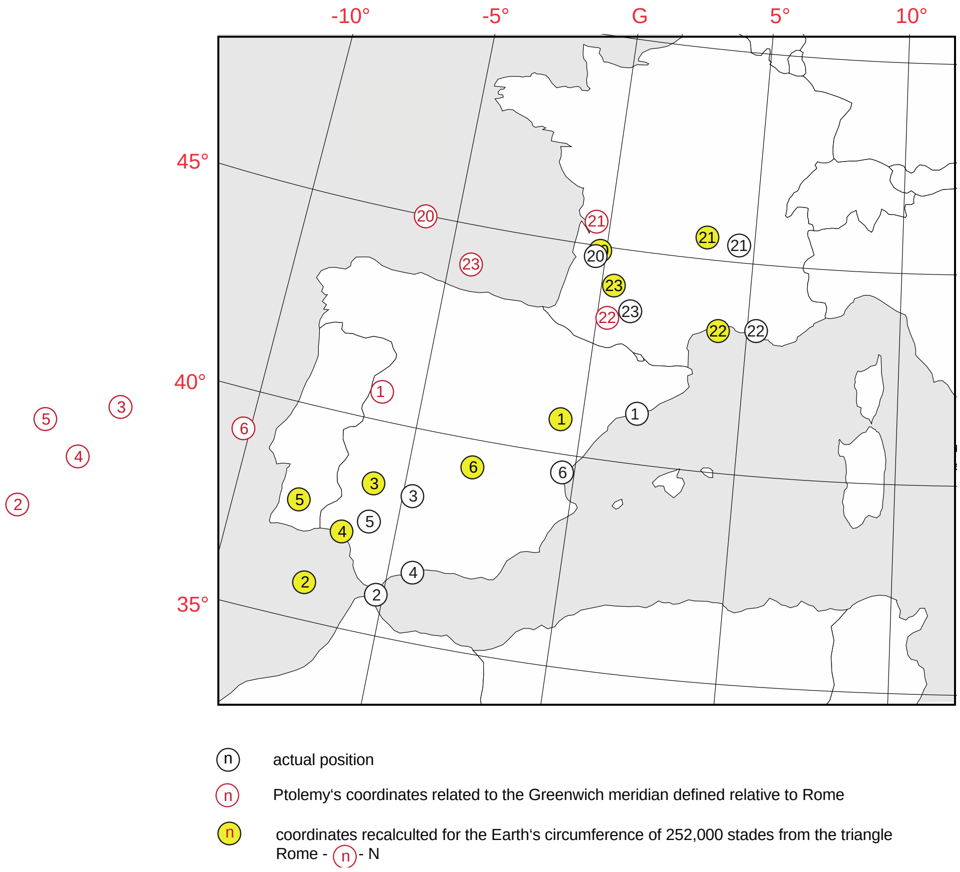

Figure 3. Recalculation of some prominent locations in the western Mediterranean for the circumference of the Earth equal to 252,000 stades. The reference point for this recalculation is Rome.

A thorough investigation of some local Ptolemaic maps has recently being applied for identification of the locations given in *Geography* by statistical analysis based on the Gauss-Markov model.[4] Unfortunately, the authors have taken the elongation of Ptolemy' world map as *a priori* and also seem to have attributed as the main error the use of different values of the stade in ancient sources (Kleineberg et al. 2012, p. 13.). In our treatment, the simple transformation of the Ptolemaic coordinates to the circumference of the Earth measured by Eratosthenes drastically improves the positions of

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[4] Kleineberg et al. 2010; Kleineberg et al. 2012.

Ptolemaic locations – under presupposition that both scholars used the same length of the stade. It is clear that due to enormous scope of information which Ptolemy had to assimilate in his geographical treatise some pollution caused by different measuring units was unavoidable. Nevertheless, it is also clear that both authors used in principle the same stade and the excessive distortion of Ptolemy´s world word is due to a wrongly conceived standardised degree. Our thesis presents an easy explanation and a coherent correction for this.

3. Conclusion

The results presented in this text have not, at first, aimed at obtaining a new estimation of the length of the stade used by Eratosthenes or by Ptolemy. Being rather motivated by a purely mathematical problem, i.e. recalculating the spherical coordinates given at the sphere with the circumference of 180,000 units to the spherical coordinates at the sphere with the circumference of 252,000 units, we have obtained results which show that the excessive distortion of Ptolemy's world map can be easily explained under the presupposition that the length of the stade used by Ptolemy coincides with that of Eratosthenes. In fact, Ptolemy's world map is distorted in a very special way: the latitudes of the known localities coincide approximately with their actual values whereas the longitudes show an excessive distortion along the east-west direction. Our recalculations show that such a distortion appears due to an erroneously adopted value for the circumference of the Earth or, equivalently, for the length of 1° = 500 stades at used by Ptolemy instead of 1°= 700 stades as given by Eratosthenes.

The first results for the Mediterranean region show that if Ptolemy would have adopted Eratosthenes's value, his map would have an unexpectedly high level of precision, in many cases, the longitudinal errors would be comparable with the latitudinal ones. As a consequence, by linking the recalculated positions of the identified localities with their actual positions, we can confirm a very high precision of Eratosthenes' result for the circumference of the Earth.

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